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PROCEEDINGS

## Fourth Annual Logistics Conference

PART II - Restricted Section

Jointly sponsored by

THE GEORGE WASHINGTON UNIVERSITY

Logistics Research Project

and

DEPARTMENT OF THE NAVY

Office of Naval Research

Held at the General Services Administration Auditorium

Washington, D. C.

16, 17, 18, 19 March 1953

Logistics Research Project

WASHINGTON, D. C.

1953

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PROCEEDINGS  
FOURTH ANNUAL LOGISTICS CONFERENCE

PART II -- Restricted Section

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## Preface

It is with pleasure that we make available copies of the Proceedings of the Fourth Annual Logistics Conference. We hope that the four annual logistics conferences, beginning with a working conference of limited attendance in 1950, have served the purposes of stimulating fundamental and applied research in the field of logistics and disseminating useful information concerning the role of logistics in military planning and operations.

The Fourth Annual Logistics Conference contained papers of various security classifications. For the expedition of the distribution of the Proceedings, and for the convenience of the recipients of copies thereof, the Proceedings have been bound and distributed in four parts. It is hoped that this procedure will enhance the value of the Conference papers to the users.

We express our sincere appreciation of the active collaboration of extremely busy people in making the Fourth Annual Logistics Conference a success. We are indebted to the speakers for the interesting and important talks they prepared and delivered. We thank The George Washington University and the Office of Naval Research for their assistance in the Conference arrangements. And last, but in no sense least, is our expression of thanks to all those who attended the Conference and whose participation made it worthwhile.

E. W. Cannon  
Principal Investigator  
Logistics Research Project

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## PROGRAM

General Chairman: Dr. Mina Rees

*Director, Mathematical Sciences Division, Office of Naval Research*

Chairman of "Practical" Sessions: Rear Admiral Henry E. Eccles, USN, Retired

Chairman of "Theoretical" Sessions: Professor Oskar Morgenstern

*Princeton University*

Monday, 16 March 1953  
Planning Problems and Techniques  
("Practical" Session, 1)

### Welcoming Remarks

Dr. Mina Rees,

*Director, Mathematical Sciences Division, Office of Naval Research*

Rear Admiral C. M. Bolster, USN

*Chief, Office of Naval Research*

Dean Martin A. Mason

*The George Washington University*

### Introduction to the "Practical" Sessions

Rear Admiral Henry E. Eccles, USN, Retired

#### 1. Fleet Logistics

Speaker: Vice Admiral F. C. Denebrink, USN

*Commander, Military Sea Transportation Service*

Formal Discussant: Rear Admiral J. E. Maher, USN

*Commander, Service Force Atlantic Fleet*

#### 2. Supply in the Pacific Theater

Speaker: Capt. O. P. Lattu, SC, USN

*Commanding Officer, Naval Supply Depot, Newport, R.I.*

Formal Discussant: Capt. J. D. Parks, SC, USN

*Service Force, Atlantic Fleet*

#### 3. Maintenance and Repair in the Fleet

Speaker: Rear Admiral W. D. Leggett, Jr., USN

*Deputy Chief, Bureau of Ships*

Formal Discussant: Rear Admiral W. H. Hague, USN

*Commandant, Industrial College of the Armed Forces*

Tuesday, 17 March 1953  
Logistics Data Processing  
("Practical" Session, 2)

1. Survey of Modern Methods of Data Processing  
Dr. E. W. Cannon  
*Principal Investigator, Logistics Research Project*
2. Data Storage Devices and Techniques  
Jacob Rabinow  
*National Bureau of Standards*
3. Digital Input and Output Devices for Automatic Computers  
Dr. Nelson M. Blachman  
*Office of Naval Research*
4. A Description of the Logistics Computer  
Lt. R. J. Rossheim, USNR  
*Office of Naval Research*
5. Applications of UNIVAC to Air Force Programming Problems  
Paul Schell  
*Headquarters, United States Air Force*
6. Application of CRC Computer to Bureau of Aeronautics Problems  
D. O. Larson  
*Bureau of Aeronautics*
7. Application of the Logistics Computer to Naval Logistics Problems  
J. Jay Wolf  
*Logistics Research Project*

Wednesday, 18 March 1953  
Distribution Control  
("Theoretical" Session, 1)

Introduction to the "Theoretical" Sessions  
Professor Oskar Morgenstern  
*Princeton University*

1. Optimal Technology for Supply Management  
Speaker: Rear Admiral Frederick L. Hetter, SC, USN  
*Bureau of Supplies and Accounts*  
Formal Discussant: P. F. Hilbert  
*Office of the Air Controller*
2. The Limits of Centralization  
Speaker: Dr. T. M. Whitin  
*Princeton University*  
Formal Discussant: Prof. M. E. Salveson  
*University of California*

3. Estimating Shipping Requirements at Short Range  
Dr. Harry M. Hughes  
*University of California*
4. Optimal Scheduling in Transportation  
Speaker: Dr. I. Heller  
*Logistics Research Project*  
Formal Discussant: Martin Shubic  
(of Papers 3 & 4) *Princeton University*

Thursday, 19 March 1953  
Theory of Games  
("Theoretical" Session, 2)

1. The Solution of Games by Behavior Strategies  
Speaker: Dr. H. W. Kuhn  
*Bryn Mawr College*  
Formal Discussant: Gerald Thompson  
*Princeton University*
2. Reduction of Games in Extensive Form  
Speaker: Dr. Norman Dalkey  
*The Rand Corporation*  
Formal Discussant: Lloyd Shapley  
*Princeton University*
3. Blotto-Type Games  
Speaker: Dr. D. W. Blackett  
*Princeton University*  
Formal Discussant: Dr. W. H. Marlow  
*Logistics Research Project*
4. Machine Representation of a Symmetric Air War Game  
BrigGen L. I. Davis, USAF  
*Air Research and Development Command*



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Armed Services Textile & Apparel  
Purchasing Agency  
New York, New York

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STELINFELD, L. E.  
U. S. Coast Guard  
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The Johns Hopkins University  
Chevy Chase, Maryland

SYRAN, A. G.  
Mutual Security Agency  
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TERRY, CHARLES  
Naval Security Station  
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Institute for Air Weapons Research  
University of Chicago  
Chicago, Illinois

THOMPSON, GERALD  
Princeton University  
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Arlington, Virginia

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University of Michigan  
Ann Arbor, Michigan

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National Bureau of Standards  
University of California  
Los Angeles, California

TORRANCE, C. C.  
U. S. Naval Postgraduate School  
Monterey, California

TUCKER, A. W.  
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TUCKER, J. F., Capt., USN  
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Mechanicsburg, Pennsylvania

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Engineering School  
Fort Belvoir, Virginia

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Norfolk Naval Shipyard  
Portsmouth, Virginia

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Newport, Rhode Island

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WOOD, R. M., Lt. Col., USMC

Marine Corps Schools  
Quantico, Virginia

WOODBURY, M. A.

University of Pennsylvania  
Philadelphia, Pennsylvania

WRIGHT, J. M. P., Capt., USN

Bureau of Ordnance  
Washington, D. C.

YOUNG, W. M.

Princeton University  
Princeton, New Jersey

ZIMET, S., Cdr., USN

Bureau of Supplies and Accounts  
Washington, D. C.

## WELCOMING REMARKS

by

The General Chairman, Dr. Mina Rees  
Director, Mathematical Sciences Division  
Office of Naval Research

This is the Fourth Annual Logistics Conference sponsored by the Office of Naval Research and The George Washington University. The first of these conferences was planned soon after ONR began its participation in the development of this relatively new art and science of logistics. We feel that one of the special needs is to provide for the exchange of the ideas and experiences of the diverse groups engaged in related work. It is in this way that scientific progress has been achieved across the ages and it is essential that the ideas that are generated in one group be subjected to the criticism, harsh or otherwise, of other groups. Thus they will be shaped into a form that may be good and useful. Those of us who are engaged on the civilian front in work related to logistics are eager that those of you who have large military experience should bring your experience and judgment and criticism, adverse or favorable, to play upon the ideas which we generate. In this particular field there are so many groups, military and civilian, working under the restriction of military classification, that the problem of interchange and criticism is particularly difficult. It is in this general area that we hope this conference will be effective. The George Washington University Project is only one facet of the ONR program in logistics. This program is concerned both with short range and with long range work and there are many other aspects of the program besides those which are represented at The George Washington University. It is particularly critical, however, to secure an interchange of judgment and criticism in the area of the short range program, and it is in this area that we hope to make progress during the first two days of the conference. The obverse of the picture is this: there is a need for some awareness of the rather long-haired work on the part of the practical workman. This is so partly because it can provide the research worker with an orientation in the selection of significant and fruitful directions for his research; partly because it can facilitate the translation into use at the very earliest possible time of any useful results which come out of the research. And so we hope that, although some of the papers on the last two days may seem a little remote, many of you will understand and participate in the discussions. It may be worth noting that the opening speaker of the so-called "theoretical" session will be Admiral Hetter who can hardly be accused of being too far removed from the scene of operations. The first two days are devoted to the so-called "practical" session and deal with planning problems and techniques and with logistics data processing. I am sure that you are all aware that the new Logistics Computer has just been delivered and will be on display during the conference. The second two days are devoted to the "theoretical" session and the topics will be distribution control and the theory of games. Before we proceed with our program, I am privileged to introduce representatives of the two organizations sponsoring this conference so that they may bring you greetings from their organizations. Rear Admiral Bolster, Chief of Naval Research, has been associated with the ONR Logistics Program since its inception and has given us support at every turn. I am particularly happy and privileged to present Admiral Bolster.

## WELCOMING REMARKS

by

Rear Admiral C. M. Bolster, USN  
Chief of Naval Research

Ladies and Gentlemen:

It is certainly an inspiration to all of us to see such a fine group here this morning and particularly to note that there is such a strong representation from the personnel of the Army, Navy and Air Force. As Dr. Rees has told you, we have been working very hard for a long time on this general problem of logistics, particularly that of bringing the many contributions of mathematics and science to bear in order to arrive at a more effective method of handling such problems. We have spent many hours both on the theoretical and practical side of this problem, since we all are extremely anxious to make a real contribution to the users of logistics services.

I want to welcome all of you and to thank you for coming here this morning, for your presence will help greatly in making this Symposium a success. It is through the efforts of you people, many of whom I know from personal knowledge are extremely busy with other things, that we achieve the understanding and progress so necessary for this effort. We appreciate the effort of those who have prepared papers which will be presented here. We also wish to express our thanks to those who have taken the time to prepare formal comments. For example, I have been working with Admiral Leggett on a special board, and I frankly do not see how he has had time to prepare his paper. On the other hand, I have been told that it is going to be a very fine paper which we are very grateful to have him present. As you look at the list of papers and speakers here today, you are struck by the wide variety of experience and background which they represent. For the first time, I believe we will be getting a lot of the flavor of real operating experience, and as Dr. Rees has said, the thing that makes such a project really successful is to be sure that it reflects the true needs of the services and not merely some theoretical problem.

As I said earlier, this project has been under way some time, and we have all looked forward to the period when we would have a computer available with which we could test some of the theories being developed. This computer is here in Washington now, and we are eagerly anticipating the demonstrations that will be possible on it, to see just how it will really solve the problems when you feed the right numbers into it. I, personally, have great confidence that it will do all the things claimed for it, simply because of my belief and great confidence in the people doing the work. Certainly, we couldn't have a finer group of people than those who have been working on this project and on the computer.

I want again to welcome you and to say that we in ONR are extremely proud to participate in this joint effort with you.

WELCOMING REMARKS  
by  
Dean Martin A. Mason  
The George Washington University

Doctor Rees, ladies and gentlemen:

One of the pleasant things that a Dean has to do is from time to time to bring words of welcome from his University to those people whom the University serves. I was particularly happy to have this opportunity given to me to greet you people and make you welcome so far as I can, first because the University family has a deep and appreciative interest in the problems of supply or logistics, and secondly because I have had in my professional career some experience with logistics, insofar as it applies to military operations.

I have the feeling that there are three elements which characterize modern logistics. One might be called the trappings (those things that are luxuries in combat), and one might be called impedimenta (those things someone thinks are necessary); and the last is that body of necessary things vital to the success of a military operation. A few months ago I had occasion to talk to an underwater swimmer, a member of a Navy underwater demolition team, and I asked him what he really needed in order to do his job. It did not take him long to figure out the answer, and he did not take many words to tell me the answer. He just said, "Air, guts, and a gismo."

Now this probably reduces the logistics problem to its simplest terms, but these were terms which I, as an engineer, could understand. It appeared to me that if logistics problems could be solved with as much simplicity as this hardy character brought to his problem, perhaps there would not be quite so much need for computers, and money, and the large assembly of brains and talent that we have here.

So I like to think of logistics then, in terms of "air, guts and a gismo." I don't know all that you are concerned with, but I am sure that you are going to put the problems and solutions that you have to work with in much more elegant language. I doubt if they can be any clearer than those of the underwater swimmer.

The University, as I have indicated, is happy, of course, to participate in the attack on the difficult problems of logistics. At the present time many people wonder what Universities are really for, whether they are havens for Communists, or a place where long-haired people can be given the necessary where-with-all to continue to have beans and bread at least once a week, or whether they really do develop new knowledge and try to disseminate that knowledge. We have a strong feeling, of course, in our University, as every other university has, that we are trying to develop in knowledge, and to disseminate it. The field of study of logistics appears to us to offer considerable opportunity for the development of new knowledge. Certainly, all of you people know that the knowledge that exists needs to be disseminated somewhat

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better. We feel this is truly an effort in which a university might well cooperate.

We are pleased indeed, therefore, for this opportunity to join with the Office of Naval Research in working on a relatively important problem.

Our interest, as one of your joint hosts, is to see that you are comfortable, and that you feel you are among friends. You probably won't after you have been here a few days; but that is simply the atmosphere in Washington.

We are grateful to the General Services Administration for making available to us this more comfortable auditorium than the austere classrooms and facilities that we would have had at the University.

Let me state again the pleasure of the University in welcoming you to this Conference, and our great appreciation to the Office of Naval Research for asking us to join in such a conference.



## INTRODUCTION TO THE "PRACTICAL" SESSIONS

by

Rear Admiral Henry E. Eccles, USN Retired

1. The major problem facing the United States today is that of maintaining our National Security without resorting to rapid inflation. In order to do this we must develop strong, adequately supported combat forces at a minimum cost. This overall problem resolves itself into several related lesser problems:

A. The selection of weapons and weapons systems most suitable for attaining National Security and our National Objectives.

B. The determination of how these weapons and weapons systems can best be employed to attain these objectives.

C. The manner in which these weapons systems should be organized; that is to say the command relations that should be established in the Combat Forces.

D. The determination of how the forces employing these weapons can be most effectively supported.

E. The determination of how best to provide for the overall command and departmental administration of these forces.

2. In considering these problems we find many strong differences of opinion and certain startling paradoxes.

A. The differences of opinion largely stem from differences in basic philosophy of war, strategy, and the employment of weapons and forces. However, these differences are greatly aggravated and made urgent by the problem of the Budget - "The Battle for the Dollar", - which is a Logistics Problem.

B. The most startling paradox is found in the fact that the slogan of "business efficiency" is being invoked by persons advocating administrative practices which are contrary to the trend in our major businesses. At a time when some authorities are emphasizing the evils of overcentralization in Government in general, and when others consider that many of our military deficiencies stem from overcentralization, there arises a demand for still greater centralization. All the while large companies are tending toward decentralization in their management.

3. The differences in military philosophy we should accept and work out in our traditional manner by patient study and education. The paradoxes and contradictions of the demand for more centralization stem from a superficial approach to the problem and from impatience.

4. The size of any enterprise can be roughly measured by the number of its employees and its gross income. In 1951 General Motors, General Electric, American Telephone and Telegraph and U. S. Steel employed a

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total of about 1,630,000 persons. Their combined gross income was about \$16,935,000,000.

In Fiscal year 1953 the U. S. Armed Forces were composed of a total of about 4,825,000 persons, both military and civilian, and they had a budget of about \$48,600,000,000. That is to say they were about three times the size of the four industrial giants combined. In fact the Navy alone, with its 1,500,000 personnel and \$13,170,000,000 budget, is about the same size as this hypothetical industrial combination.

Now, granted that statistics can be very misleading, nevertheless these figures do give us, in terms of well known industrial concerns, the order of magnitude of the problem of military management.

5. I presume that if we attempted a corporate consolidation of General Motors, General Electric, American Tel and Tel, and U. S. Steel, and then insisted that the budget for 1954 be submitted by each division of the combined company before its budget for 1953 had been established by the five hundred man Board of Directors, there might be some areas of imperfection, and the stockholders might become impatient. Some might even say that such a corporation is unmanageable in a democracy.

6. And yet the problem of creating, employing, and supporting our combat forces must be managed, and managed with efficiency. The application of sound principles of Logistics and of Logistics planning enters into every one of the problems mentioned. In some instances it is the vital element. And yet these vital principles have not yet been adequately formulated, let alone applied. Therefore, for the overall problem to be solved, there must be patient, continuing study and research. But first the problem must be seen in its whole immense size; and the relationships that exist among the various parts of the problem must be understood.

7. The logistics aspects are themselves so great that in this conference we can consider only certain portions. After stating several of them, we will discuss certain tools that we hope may be useful in their solution. Gradually, by clear statement, and by patient discussion, our understanding may be increased. Much of what will be said during these next four days will deal with new ideas, and with the development of theory. This is, of course, the purpose of our meeting and our only hope for continued progress. However, these ideas must be based on an understanding of the facts of life.

8. Because of our great preoccupation with what happens in Washington, there may be a tendency to forget that our Logistical Establishment has, as its sole purpose, the support of the Combat Commander. We can make many minor mistakes and readily absorb them; but if we ever forget the point of view of command in the field, we will make a major mistake that can be fatal.

## INTRODUCTION TO THE "PRACTICAL" SESSIONS

9. And so today we will open our conference by a discussion of the problem of the support of the Naval Aspects of the Korean fighting. From 1945 - 1950, Vice Admiral F. C. Denebrink was engaged in various logistical tasks in the Pacific and in Washington. From 1950 to Nov. 1952 as Commander Service Force U. S. Pacific Fleet he was charged with the Logistic support of all Naval Forces in the Pacific Ocean. He is now Commander of the Military Sea Transportation Service. With this background he is particularly well suited to discuss "Fleet Logistics" from the working point of view--the point of view that we must never forget. I take great pleasure in introducing Vice Admiral Francis C. Denebrink, U. S. Navy.

## INTRODUCTION TO THE "THEORETICAL" SESSIONS

by

Prof. Oskar Morgenstern

The general chairman, Dr. Mina Rees, said that on Wednesday a person named Oskar Morgenstern would take the chair for the so-called "Theoretical Session." I am this person and I want to say that I am very honored in having this privilege of introducing so many excellent speakers.

The overall program is broken into two parts: practical and theoretical. This sort of division easily irks somebody who works in theory, because it appears as if it were impractical; but if you look carefully, you will find that the arrangers of the program have in their wisdom put these two words in quotation marks, thereby indicating that the common assumption that theory is not practical, might not be true. I hope that the speeches and the talks and other contributions and discussion will prove this.

Naturally, we will progress gradually more and more to abstract things in the sessions of today and tomorrow, but you may well be aware that, for example, if one can define an optimum operation of a system, one will really know something because one could discover whether a given system actually is near it or not and how one can approach the optimum.

Insofar as the theory of games is concerned, it appears of course, in many ways even more remote from such questions; yet on the other hand, some hardware has already been constructed with the aid of the theory. For example, certain types of planes have been selected over other types of planes precisely on the basis of this theory. Similarly, at present, for example, certain works are in progress which, if successful, will lead to the construction of a particular type of guided missile because it would be possible to build certain concepts of strategy into the hardware itself. So it is quite clear that there are very direct practical connections.

The talks of this morning begin with the talk on "Optimal Technology for Supply Management." The speaker is Rear Admiral Frederick L. Hetter of the U. S. Navy; he has a wide experience in the Atlantic and Pacific, and since January 1951 he has been Assistant Chief of the Bureau of Supplies and Accounts. It gives me great pleasure to introduce Rear Admiral Hetter.

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## BLOTTO-TYPE GAMES

by

Dr. D. W. Blackett  
Princeton University

### 1. The Problem of Colonel Blotto

In one version of the problem of Colonel Blotto, the colonel and the enemy commander divide their forces among certain forts without knowing the opposing disposition. The two commanders are in charge of the forces of two unfriendly nations on the night that war is declared. In each of the passes along the common frontier is a fort, valuable in war but unoccupied in peace. On the night war is declared both Colonel Blotto and his opponent deploy their troops under the cover of darkness in the various passes in preparation for battles for the frontier forts. If one side has more units of troops at a particular fort, that side will capture the fort and the opposing troops. If both commanders send the same number of units to the same fort, neither side will capture the fort or take prisoners. Each commander scores one point for each fort and each opposing unit captured by his troops. On the other hand each commander loses one point for each fort and each unit captured by his opponent. The problem of Colonel Blotto is to deploy his units, in ignorance of the enemy disposition, so as to make his score as large as possible. Similarly the enemy commander tries to make his score as large as possible. Since making the enemy's score as large as possible is equivalent to making the Colonel's as small as possible, the tactical situation faced by Colonel Blotto and his opponent is a two-person zero-sum game.

### 2. Blotto-Type Games

The problem concocted for Colonel Blotto has been used to show how optimal strategies solving a military problem might be found by considering the military situation as a game. Although the colonel's

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problem is artificial, the game formulated from it illustrates a general type of game which seems to have many potential applications to problems of current military interest. Specifically a game is a blotto-type game if

Two players contend in various independent operations area.

Each player has certain forces he must distribute to these operations area before he knows the disposition of opposing forces. The gain (measured numerically) for a player on a particular operations area depends only on the operations area and the forces committed to that operations area by the two players. The total payoff to a player is the sum of his gains in the individual operations area.

### 3. Military Examples of Blotto-Type Games

The definition of blotto-type games covers any military situation in which the contesting nations or commanders divide their forces among independent theaters, battlefields, or engagements without knowing the deployment of opposing forces. Examples of such situations are:

a. Bombers are allocated to attack widely separated target areas and interceptors are deployed to defend these areas without either side knowing the distribution of opposing aircraft.

Here the operations area are the different target areas and the payoff to the attacker is the expected value of damage caused by bombing plus the expected value of interceptor losses minus the expected value of bomber losses. Under the assumption that the payoffs in the different target areas are additive, the problem of assigning bombers and interceptors is a blotto-type game.

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## BLOTTO-TYPE GAMES

b. Convoys are routed without complete knowledge of submarine locations, while submarines are stationed without advance information about convoy routes.

In this example the different possible convoy routes are operations area and the payoff to the submarines is the expected value of convoy losses (including time losses) minus the expected value of submarine losses. If the convoy routes under consideration are sufficiently separated so that no one submarine can simultaneously threaten more than one of the possible routes, the operations area are independent and the problem of routing convoys is a blotto-type game.

c. Amphibious landings are made without complete information about the deployment of the defending forces, while these forces are stationed without knowledge of the landing beaches selected.

Here the operations area are the different possible landing beaches, and casualties and positions gained or lost measure the payoff.

d. If different types of bombers can deliver atomic bombs, the assignment of bombs for an atomic-bombing mission must be made in partial ignorance of how defense potential will be allocated against the different types of bombers. On the other hand this allocation must be made without knowledge of the bomb assignment.

The operations area are the different interception missions and the payoff is the expected number of atomic bombs dropped on targets. In so far as commitment of defense potential to intercepting one type of bomber detracts from the defender's ability to intercept other types, this allocation problem is a blotto-type game.

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e. If the defender's supply of ammunition along a battlefield is not unlimited, the limited supplies must be distributed among the different sectors without knowing exactly where to expect attacks. On the other hand the attacks are made in ignorance of the defender's allotment of ammunition.

This problem of logistics and tactics is a further example of a blotto-type game.

### 4. A Special Class of Blotto-Type Games

Since many military situations can be formulated as blotto-type games, it is natural to ask "What general statements can be made about solutions of blotto-type games?"

One class of games for which theorems can be proved consists of games in which

Two players (A and B) fighting in  $N$  independent operations area (labeled  $1, 2, \dots, 1, \dots, N$ ) must distribute their forces ( $F$  and  $G$  units respectively) to the operations area before discovering the opposing deployment. The payoff on operations area  $i$  (a numerical measure of the gain of A or equivalently of the loss of B) is given by a function  $P_i(x, y)$  depending only on the operations area and the opposing forces  $x$  and  $y$  committed to that operations area by A and B respectively. The payoff of the game as a whole is the sum of the payoffs in the individual operations area.

The functions  $P_i(x, y)$  are assumed

I $\alpha$ . Convex in  $x$  for all  $i$  and  $y$  or

$\beta$  Concave in  $x$  for all  $i$  and  $y$



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## BLOTTO-TYPE GAMES

IIa. Concave in  $y$  for all  $i$  and  $x$  or

IF Convex in  $y$  for all  $i$  and  $x$

### 5. The Significance of Assumptions I and II

A restatement of Condition Ia is that the payoff to A in every operations area satisfies a law of increasing returns in terms of units committed by A against fixed opposition by B. This is the case if increasing the number of A's units overloads the defense of B so that the payoff to A per unit committed increases with the number of units. An example in which there is such an increase in effectiveness per unit is a situation in which two submarines making a coordinated attack on a convoy would sink more than twice as many ships as one could sink operating alone.

Condition IF means that in every operations area the payoff to A against fixed opposition by B satisfies a law of diminishing returns. This condition would hold if an increase in the number of units sent to battle by A leads to sufficient overkilling of B's forces so that each additional unit committed by A decreases the average effectiveness of A's units. An example in which the law of diminishing returns applies is an attack on a bomber with guided missiles. The more missiles dispatched the greater the probability of killing the bomber, but the less the kill probability per missile expended. This is because of the wastage of missiles which would result if more than one missile did lethal damage to the bomber.

Under Condition IIa the loss of B (i.e. the gain of A) per unit committed by B decreases as the number of units increases. In other words the gain of B satisfies a law of increasing returns.

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Under Condition II $\beta$  the loss of  $B$  per unit sent to battle increases as the number of units increases. Equivalently the gain of  $B$  satisfies a law of diminishing returns.

In summary, Assumptions I and II may be rephrased

- I $\alpha$ . In each operations area the gain of  $A$  against fixed opposition satisfies a law of increasing returns, or
- $\beta$ . In each operations area the gain of  $A$  against fixed opposition satisfies a law of diminishing returns.
- II $\alpha$ . In each operations area the gain of  $B$  against fixed opposition satisfies a law of increasing returns, or
- $\beta$ . In each operations area the gain of  $B$  against fixed opposition satisfies a law of diminishing returns.

### 6. Optimal Strategies under Assumptions I and II

When Condition I $\alpha$  holds, there is an essentially-unique optimal mixed strategy for  $A$  which sends either all or none of his forces to a particular operations area. The optimal strategy must be such an all-or-nothing strategy because among all mixed strategies sending the same expected number of units to the operations area the one which never divides forces is the best for  $A$ . This is a reflection of the geometric fact that when  $P_1(x,y)$  is a convex function of  $x$ , the graph of  $P_1(x,y)$  as a function of  $x$  is below the chord joining its endpoints.

Under Condition I $\beta$  there is an essentially-unique optimal strategy for  $A$  which uses a single fixed deployment of forces. The optimal strategy is pure because among all mixed strategies sending the same expected number of units to an operations area the one which sends exactly that number with probability one is the best for  $A$ . This reflects the

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## BLOTTO-TYPE GAMES

fact that when  $P_1(x,y)$  is concave in  $x$ , the graph of  $P_1(x,y)$  as a function of  $x$  is below every tangent to the graph.

Under Condition II $\alpha$  the optimal strategy for B is pure since the graph of  $P_1(x,y)$  as a function of  $y$  is above each of its tangents.

When Condition II $\beta$  holds, the optimal strategy for B is an all-or-nothing strategy sending all or none of his forces to a particular operations area. This is because the graph of  $P_1(x,y)$  as a function of  $y$  is above the chord joining its endpoints.

### 7. The Calculation of Optimal Strategies

Case  $\alpha\beta$ : Conditions I $\alpha$  and II $\beta$  hold.

In this case the following theorem gives a convenient set of equations whose solution (if there is a solution) determines both the optimal all-or-nothing strategy of A and the optimal pure strategy of B.

Theorem: If  $y_1, \dots, y_N, q_1, \dots, q_N$  are non-negative numbers solving the equations

$$P_1(F, y_1) - P_1(0, y_1) - P_2(F, y_2) - P_2(0, y_2) - \dots - P_N(F, y_N) - P_N(0, y_N) = 0$$

$$y_1 + y_2 + \dots + y_N = G \quad (1)$$

$$q_1 \left[ \frac{\partial P_1(F, y)}{\partial y} \right]_{y=y_1} - (1 - q_1) \left[ \frac{\partial P_1(0, y)}{\partial y} \right]_{y=y_1} - \dots$$

$$- q_N \left[ \frac{\partial P_N(F, y)}{\partial y} \right]_{y=y_N} - (1 - q_N) \left[ \frac{\partial P_N(0, y)}{\partial y} \right]_{y=y_N} = 0 \quad (2)$$

$$q_1 + \dots + q_N = 1,$$

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then the pure strategy sending  $y_1$  units to operations area 1,  $y_2$  to operations area 2, ..., and  $y_N$  to operations area N is optimal for B; while the all-or-nothing strategy sending all F units to operations area 1 with probability  $q_1$  is optimal for A.

An interesting specialization of this theorem occurs when there is gain or loss to either A or B only in those operations area to which A has sent forces. An example of this special situation may occur if A is routing a convoy, B is threatening the convoy with submarines, and the operations area is possible convoy routes. Under these circumstances the convoy can lose ships only on the convoy route actually used.

In the special case the equations (1) become

$$P_1(F, y_1) = P_2(F, y_2) = \dots = P_N(F, y_N)$$

$$y_1 + y_2 + \dots + y_N = G.$$

These equations show that the optimal pure strategy for B divides his forces so that when A sends all F units to a single operations area the payoff does not depend on the operations area. In a convoy problem this means the submarines should be stationed so that the expected convoy losses do not depend on the convoy route selected. Hence the submarines are optimally stationed if each convoy route is equally threatened.

A further consequence of the specialization is that the equations (2) become

$$q_1 \left[ \frac{\partial P_1(F, y)}{\partial y} \right]_{y=y_1} \dots q_N \left[ \frac{\partial P(F, y)}{\partial y} \right]_{y=y_N}$$

$$q_1 + \dots + q_N = 1$$

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Hence the optimal all-or-nothing strategy for A sends his forces to the different operations area with probabilities inversely proportional to the derivatives of the functions  $P_i(F,y)$  (i.e. inversely proportional to certain marginal payoffs) at the values of  $y$  which give the optimal pure strategy for B. The effect of this choice of probabilities is that B is penalized for any deviation from optimal play.

Case  $\alpha$ : Conditions Ia and II $\alpha$  hold.

The theorem below shows that the optimal all-or-nothing strategies for A and B may be found by solving two sets of linear equations.

Theorem: If  $q_1, \dots, q_N$  and  $s_1, \dots, s_N$  are non-negative numbers solving the linear equations

$$q_1[P_1(F,G) - P_1(F,0)] + (1-q_1)[P_1(0,G) - P_1(0,0)] = \dots$$

$$q_N[P_N(F,G) - P_N(F,0)] + (1-q_N)[P_N(0,G) - P_N(0,0)]$$

$$q_1 + \dots + q_N = 1$$

$$s_1[P_1(F,G) - P_1(0,G)] + (1-s_1)[P_1(F,0) - P_1(0,0)] = \dots$$

$$s_N[P_N(F,G) - P_N(0,G)] + (1-s_N)[P_N(F,0) - P_N(0,0)]$$

$$s_1 + \dots + s_N = 1,$$

then the optimal all-or-nothing strategy for A sends his  $F$  units to operations area  $i$  with probability  $q_i$ , while the optimal all-or-nothing strategy for B sends his  $G$  units to operations area  $i$  with probability  $s_i$ .

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If A and B gain or lose only when opposing forces meet on the same battlefield, these equations simplify to

$$q_1 P_1(F,G) + q_2 P_2(F,G) = \dots = q_N P_N(F,G)$$

$$q_1 + q_2 + \dots + q_N = 1$$

$$s_1 P_1(F,G) = s_2 P_2(F,G) = \dots = s_N P_N(F,G)$$

$$s_1 + s_2 + \dots + s_N = 1$$

Since the equations for the  $s_i$  are the same as those for the  $q_i$ , the optimal all-or-nothing strategies for the opposing players send their forces to the different operations area with the same probabilities. The effect of either of the opposing strategies may be interpreted either as threatening the opposing forces equally no matter what operations area is contested or as penalizing the opposition for any deviation from optimal tactics.

Case  $\alpha\alpha$ : Conditions I $\alpha$  and II $\alpha$  hold.

This case is the same as Case  $\alpha\alpha$  except that the roles of A and B have been interchanged.

Case  $\beta\beta$ : Conditions I $\beta$  and II $\beta$  hold.

In this case when both players have optimal pure strategies, these strategies may be found by locating the saddlepoint of the total payoff function. In general no such simple relations as the equations for the preceding cases can be given to determine such a saddlepoint.

8. The Relation of the Four Cases to Military Problems

The mathematical breakdown into cases has its parallel in

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military tactics. The analytic question whether the payoff is convex or concave in the various operations area corresponds to the military question whether forces should be divided or used as a single unit. For many types of military operations, experience seems to have answered the question. For instance ships are safer in one large convoy rather than in several small convoys. On the other hand there is some uncertainty when submarines should operate independently and when in wolf packs.

Once it has been decided whether forces should be divided or not the military problem remains: if they are to be divided, how are they to be divided, and if they are not to be divided, where should they be employed. It is the solution of this problem that may be aided by computing optimal strategies from the equations above.

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FORMAL DISCUSSION ON  
DR. BLACKETT'S PAPER  
by  
Dr. W. H. Marlow  
Logistics Research Project

1. Introduction.

The purpose of this discussion is to describe some games possessing interest as probabilistic models of situations related to warfare. These games will be illustrated by a numerical example to which one of Dr. Blackett's results applies. It should be noted that this discussion is simply a general indication of interests shared by several persons associated with the Logistics Research Project; the situation described below in 3 and the resulting game have been selected for exposition only - as examples they do not present a picture of current Project studies in the theory of games, either theoretical or computational.

2. Attrition Functions and Attrition Games.

An attrition function expresses a probability associated with the "survival" of  $x_1, \dots, x_m$  "units" if contestants 1,  $\dots$ ,  $m$  have assigned  $\xi_1, \dots, \xi_m$  "units" to some contest. Of special interest are functions dependent upon the time duration of the action and where, at a given time, the survival of a unit depends upon the number of enemy units and the number of friendly units currently engaged. An attrition game is one for which the description of the result of a single move is given by an attrition function. This contrasts with the direct specification of a payoff and there clearly are advantages in introducing fundamental lethal processes from which payoffs are derived. Study of such games (which would be of the Blotto type if they were fights for geographical positions) would be an attempt to recognize probabilistic realities and the results, both qualitative and quantitative, would be expected to include sufficient initial conditions for prescribed expectations.

3. Numerical Example.

There are two opponents, Player I having two ships and two units of aircraft, and Player II with four aircraft units. The objective of I is to unload each of his ships at one of two possible points under the opposition of II's aircraft. Each player assigns his forces in ignorance of the other. It is further assumed that the above is to be repeated, say daily, and that on a given day, no changes from the initial assignments for that day are possible for either player: for example, if units of II's aircraft are sent to a point to which no ship has been sent, these aircraft are not able to go to the other unloading point.

An example of a simple attrition function for the above situation would be:

$p_n(i, j) =$  the common survival probability of a ship assigned to place  $n$  if  $i$  of I's aircraft oppose  $j$  of II's at place  $n$ .



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For the numbers selected above, assume:

$$p_1(i, j) = (i - j + 7)/10;$$

$$p_2(i, j) = (i - j + 6)/10 .$$

Thus, place 1 is more favorable to Player I than is place 2 to the extent that say, 1 has a unit of I's aircraft added as a permanent fixture.

An example of a zero-sum game based upon the above would be one having the expected number of surviving ships as payoff to I and the negative of this number as payoff to II. This number would be the average outcome corresponding to a specified set of assignments by I and II. If the payoffs are computed by means of standard binomial expressions, it develops that certain assignments will never be made by I since he could always do better no matter what assignment is made by II. For example, I will not assign both aircraft units to place 1 and both ships to 2. After these considerations, of the nine possibilities open to I, five remain:

- F<sub>1</sub>: Assign everything to place 1;
- F<sub>2</sub>: Divide the ships, send all aircraft to 1;
- F<sub>3</sub>: Divide equally between 1 and 2;
- F<sub>4</sub>: Divide the ships, send all aircraft to 2;
- F<sub>5</sub>: Assign everything to 2.

Five possibilities are open to II: E<sub>1</sub>, E<sub>2</sub>, ..., E<sub>5</sub> corresponding to his sending, respectively, 4, 3, ..., 0 units to place 1 and the remainder of place 2.

The expected number of ships surviving under these assignments are given in the following table:

	E <sub>1</sub>	E <sub>2</sub>	E <sub>3</sub>	F <sub>4</sub>	E <sub>5</sub>
F <sub>1</sub>	1.0	1.2	1.4	1.6	2.0
F <sub>2</sub>	1.5	1.1	1.1	1.1	1.2
F <sub>3</sub>	1.4	1.1	1.1	1.1	1.3
F <sub>4</sub>	1.3	1.1	1.1	1.1	1.4
F <sub>5</sub>	2.0	1.4	1.2	1.0	0.8

The matrix game with this payoff has the following mixed strategy solution: Player I should play only F<sub>1</sub> and F<sub>4</sub> and these with equal

## FORMAL DISCUSSION ON BLACKETT'S PAPER

probability while Player II should play  $E_2$  three times as often as  $E_4$  and should play only these strategies. The value of the game is 1.3 which means that this will be the average number of ships surviving: in ten days say, Player I would expect to unload about 13 shiploads of the 20 he assigned. Further, the above means that either player can expect to be penalized for deviating from an optimal method of play. As one example - if II elects to constantly divide his aircraft equally ( $F_3$ ), I may be expected to deduce this and realize 1.4 by always sending his ships and aircraft to place 1 ( $F_1$ ).

If  $F_2$  and  $F_4$  are eliminated from the above, I may be regarded as having two homogeneous units, each composed of one ship and one aircraft unit. Then, the continuous game having as payoff function the linear extension of the former matrix falls under Dr. Blackett's Case I. Therefore, Player I has a mixed strategy while II has a pure strategy. The solution of the continuous game does correspond to the former solution since player I proceeds as before while player II, in order to realize his "pure" strategy of sending 2 1/2 units to place 1 and 1 1/2 units to 2 must alternate  $E_2$  and  $E_4$  as before.

In conclusion, the above over-simplified numerical example is intended to illustrate a Blotto game based upon fundamental rules of attrition. From the description of attrition and the chosen payoff structure the game solution and game value were computed and interpreted.

## SUMMARY OF GENERAL DISCUSSION ON

### DR. BLACKETT'S PAPER

During the general discussion concerning possible applications of the theory of games to actual military problems, the Chairman cited the Air University thesis of Col. Oliver G. Haywood, USAF, "Military Decision and the Mathematical Theory of Games". This study was described as a general comparison of principles and it was stated that copies of the thesis, reprinted as a research memorandum, might be obtainable from the Rand Corporation. Reference was also made to an abstract of the thesis in "Air University Quarterly Review," vol. 4, no. 1, 1950, pp. 17-30.

Admiral Eccles called attention to the careful studies of World War II Naval engagements being undertaken by Commodore Bates at the Naval War College. It was stated that, with proper clearances, these detailed analyses might be available for study by scientists.

The discussion concluded with several points related to the employment of chance devices in warfare. In reply to Prof. Marschak's questioning, Admiral Eccles stated that he knew of no deliberate employment of randomized mixed strategies in warfare, but he did point out the classical requirement for making decisions under slight information--in the "fog of war". In connection with procedures of a tactical nature, random devices were said to have been employed in scheduling shipping and the emplacement of certain types of weapons. Prof. Hughes added some observations based upon experiments wherein subjects displayed discernible effects in attempting to perform simple tasks in a random fashion.

## SUMMARY REMARKS

by

Chairman Morgenstern

It is 3:24 and the time table here says that at 3:29 the chairman has to make summary remarks. I think they would be somewhat anticlimactic if they were long, as they would have to be. So all I will do is to thank the various speakers, the main speakers and the formal discussants, and all those who have sat through these two days of so-called "theoretical" sessions.

I am turning the meeting over to Dr. Rees.

## CONCLUDING REMARKS

by

The General Chairman, Dr. Mina Rees

If Dr. Morgenstern's remarks would be anticlimactic, I cannot think of any that would be more anticlimactic than mine.

I wish to express, not only to the speakers, but to the Chairmen of the two sessions, the gratitude of all the participants for their excellent planning and presiding and participation.

I should like also, on behalf of the Office of Naval Research, to thank particularly The George Washington University people who have been largely responsible for the planning and carrying out of this symposium.

I mentioned in my opening remarks that one of the great needs in this new logistics game is to provide a forum for interchanging of opinion and experiences. Although I myself have been rather heavily engaged in some practical games over in the Pentagon for the last couple of days, I hear there has been a lively exchange of opinion, and some disagreement expressed on the floor, and sometimes outside. I think this is all to the good.

I want particularly to thank our military participants. I wonder if the rest of the civilians feel as I do, that we have learned a tremendous lot from what they said. Not many are here, but I would particularly like to thank General Davis, whose talk was so stimulating and worth while.

On the general front of providing a forum for interchange of opinion, I would like to take this chance to announce that the Office of Naval Research plans to initiate a journal, probably some time in the Fall, to be called the Naval Research Logistics Quarterly. It is intended that this journal should contain both theoretical and practical articles, that the writers should be drawn both from the scientific community and from the military community. You are all invited and urged to submit articles for consideration. The Journal will be carefully edited. We do not guarantee publication of anything. The objective is to provide a really adequate interchange of good work in logistics. Articles should be sent to Code 436, the Logistics Branch of the Office of Naval Research.

With these remarks I would like to declare this session of the Logistics Conference at an end, and thank all the participants, both on the floor and on the stage.